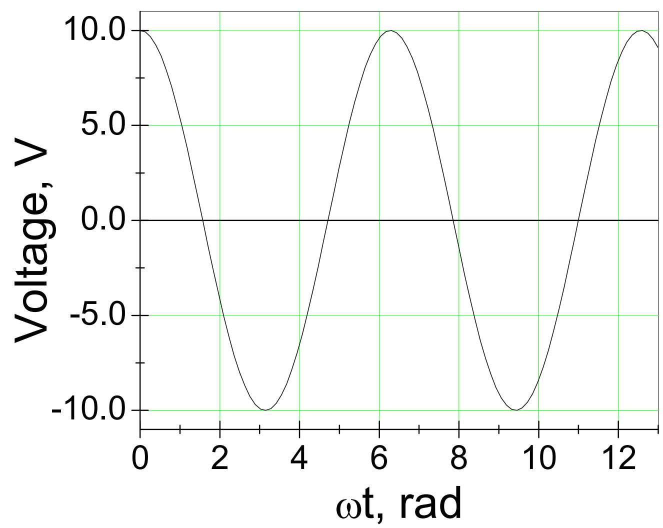
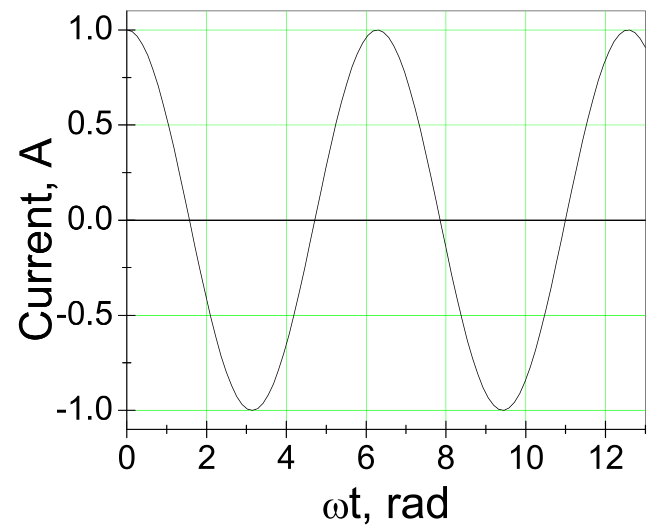
AC power through resistor

Example: i = I_M cos(ωt); I_M=1 A; R = 10 Ω ⇨ v = V_M cos(ωt); V_M = I_M*R = 10 V



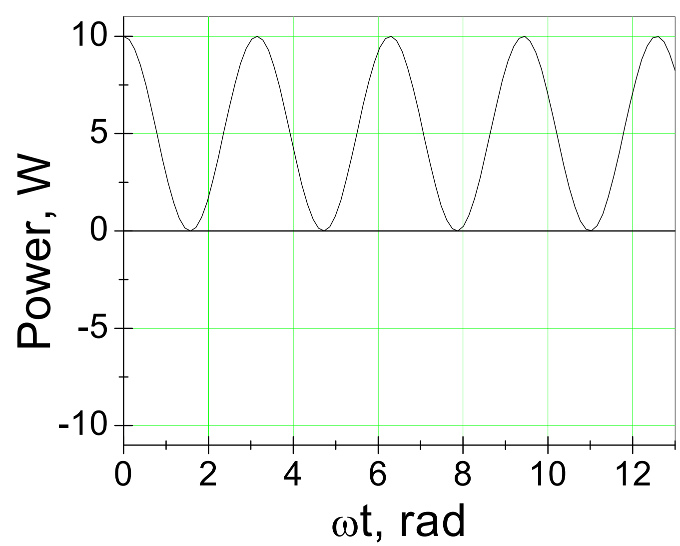


The power dissipated in the resistor:

At any point of time, *p(t) = v(t)×i(t) =*

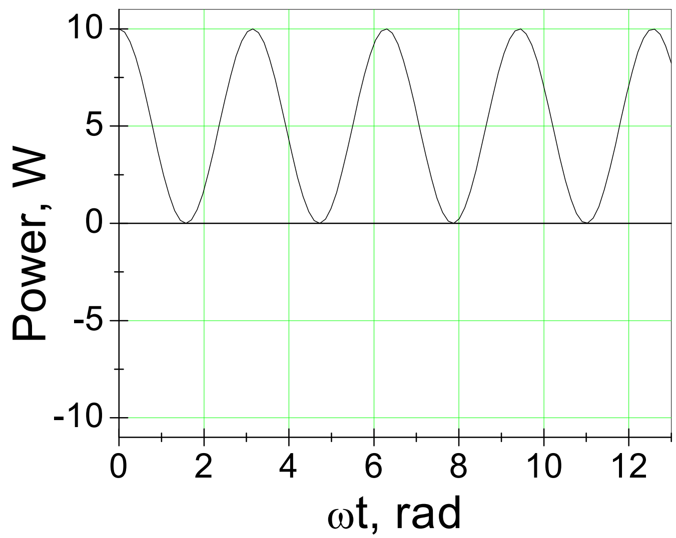
$$V_M \cos(\omega t) \times I_M \cos(\omega t) = V_M I_M \cos^2(\omega t)$$

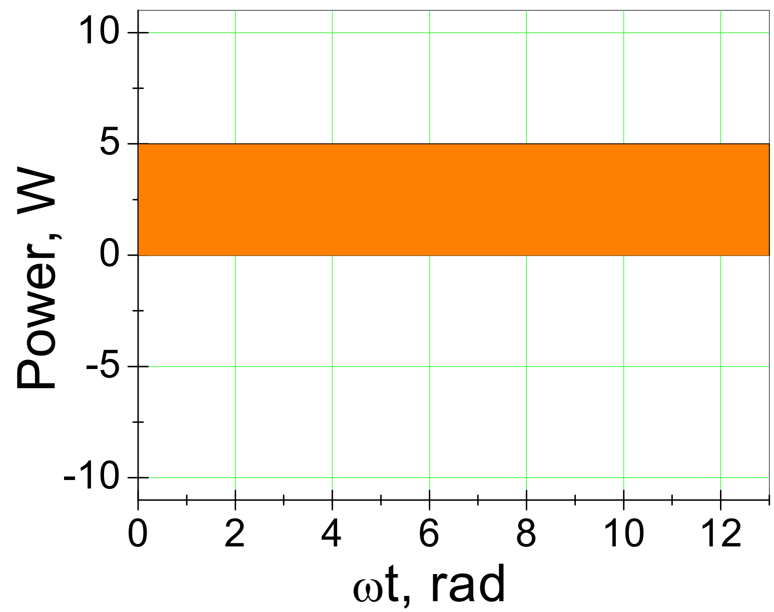
p(t) is always positive



Instantaneous and Average ("rms") powers

RMS = "root mean square" or the square root of the arithmetic mean (average)





Instantaneous power:

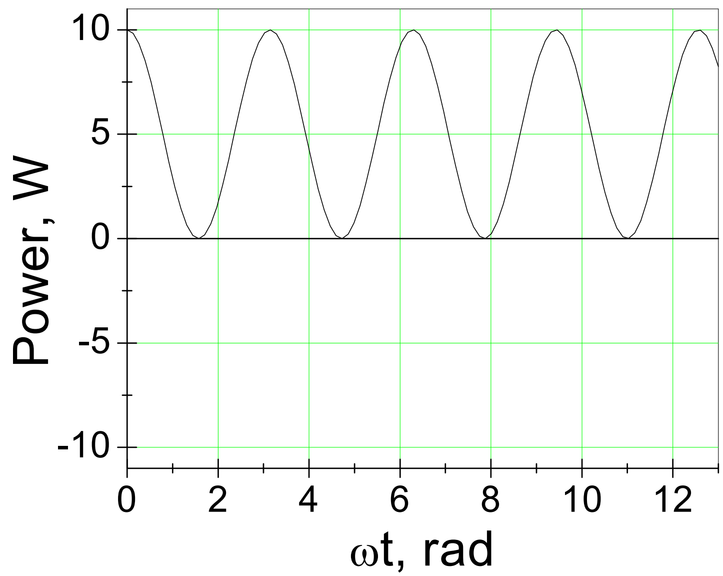
$$p(t) = v(t) \times i(t) = V_M I_M \cos^2(\omega t)$$

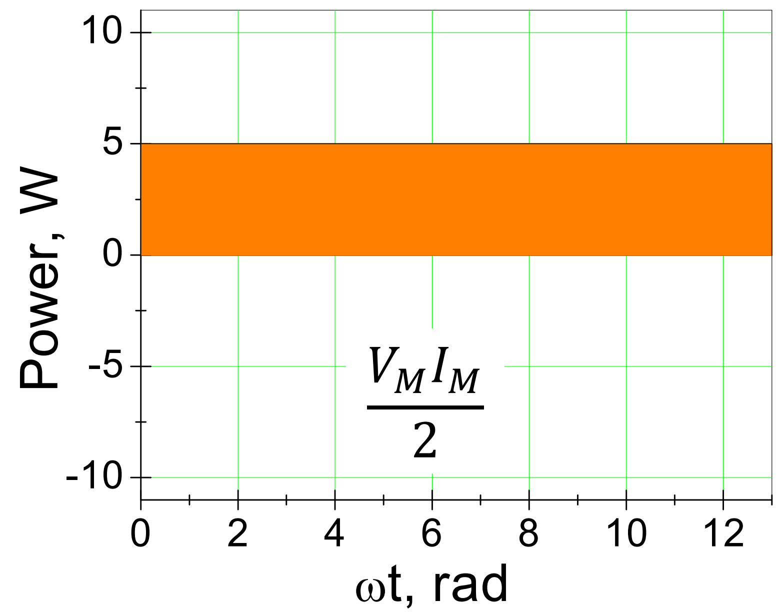
In terms of ***Joule heat dissipation***,

instantaneous power is ***equivalent*** to a certain DC power (i.e. ***time-independent***).

The equivalent DC power is called Average or "rms" power

Average ("rms") power dissipated in resistor





$$p(t) = v(t) \times i(t) = V_M I_M \cos^2(\omega t)$$

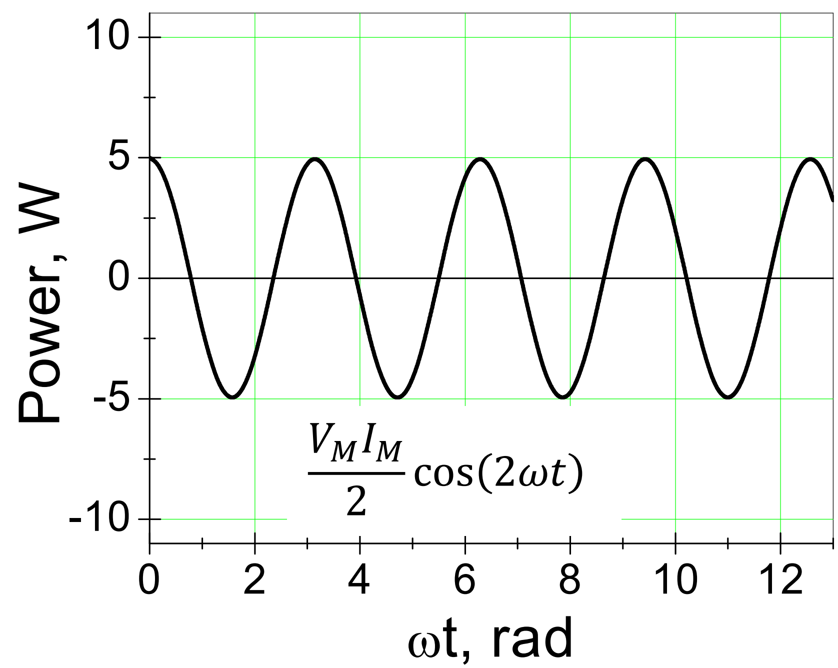
$$\cos^2(\omega t) = (1/2)\,[1 + \cos(2\,\omega t)]$$

$$p(t) = \frac{V_M I_M}{2} + \frac{V_M I_M}{2}\cos(2\omega t)$$

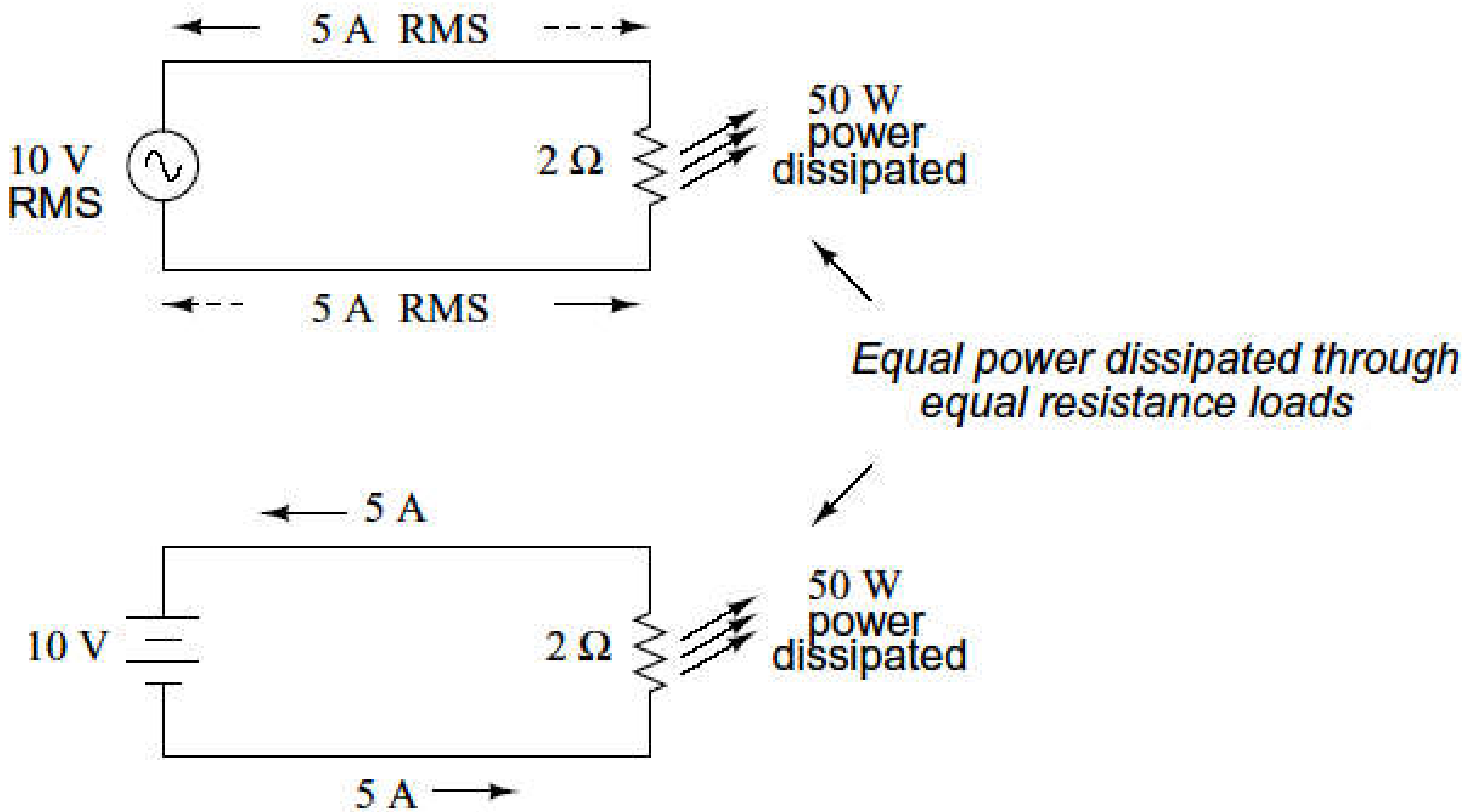
$$P_{rms} = V_M I_M / 2$$

$$V_{eff} = V_M/\sqrt{2}$$

$$I_{eff} = I_M/\sqrt{2}$$



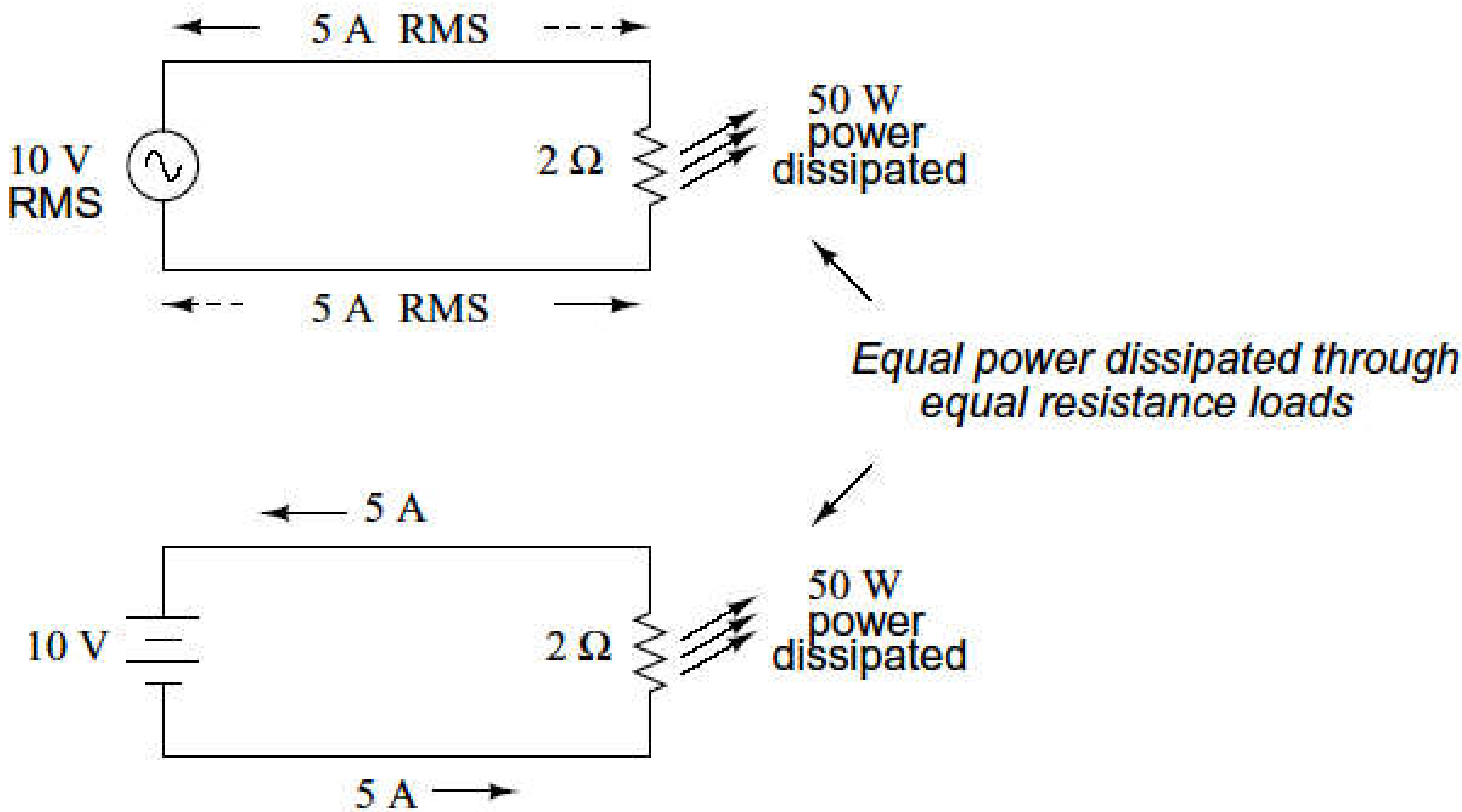
Average (RMS) characteristics of sine waves



The sine voltage source with the effective (rms) voltage 10V produces the same power as the DC battery with 10 V voltage

The equivalent DC voltage (producing the same Joule heat as the AC voltage) is called ***RMS voltage (Root Mean Square) or effective value***

Average (RMS) characteristics of sine waves



$$V_{RMS} = V_M/\sqrt{2} = 0.707V_M$$

What is the amplitude of the AC source above?

$$V_M = V_{RMS}/0.707 = 10V/0.707 = 14.14\ V$$

AC powers in R-L-C circuits

There is a phase angle between the current and voltage:

$$i = I_M \cos(\omega t);\ v = V_M \cos(\omega t + \varphi);$$

The instantaneous power:

$$p(t) = v(t) \times i(t) = V_M \cos(\omega t + \varphi) \times I_M \cos(\omega t) =$$
$$V_M I_M \cos(\omega t + \varphi) \cos(\omega t)$$

cos(a) cos(b) = (1/2) [cos(a+b) + cos(a-b)]

$a = \omega t + \varphi;\ b = \omega t;$

$$p(t) = \frac{V_M I_M}{2}\cos(\phi) + \frac{V_M I_M}{2}\cos(2\omega t + \phi)$$

cos(a+b) = cos(a) cos(b) - sin(a) sin(b)

$a = \omega t + \varphi;\ b = \omega t;$

$\cos(2\omega t + \varphi) =$
$\cos(2\omega t) \cos(\varphi) - \sin(2\omega t) \sin(\varphi)$

$$p(t) = \frac{V_M I_M}{2}\cos(\phi) \cdot [1 + \cos(2\omega t)] - \frac{V_M I_M}{2}\sin(\phi)\sin(2\omega t)$$

AC powers in R-L-C circuits

*In R-L-C circuits, the current and voltage may **both** have a phase angle with respect to the source signal:*

$$i = I_M \cos(\omega t + \varphi_i);\ v = V_M \cos(\omega t + \varphi_v);$$

We can offset the origin of the current and voltage waveforms by φ_i:

$$i = I_M \cos(\omega t);\ v = V_M \cos(\omega t + \varphi_v - \varphi_i) = V_M \cos(\omega t + \varphi);$$

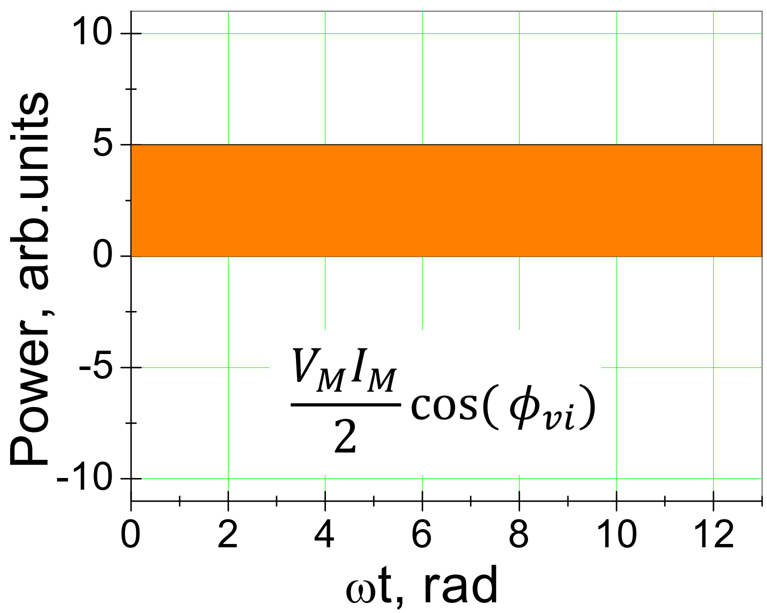
where $\varphi_{vi} = \varphi_v - \varphi_i$; *we can now use the previous expressions derived for* $\varphi_i = 0$

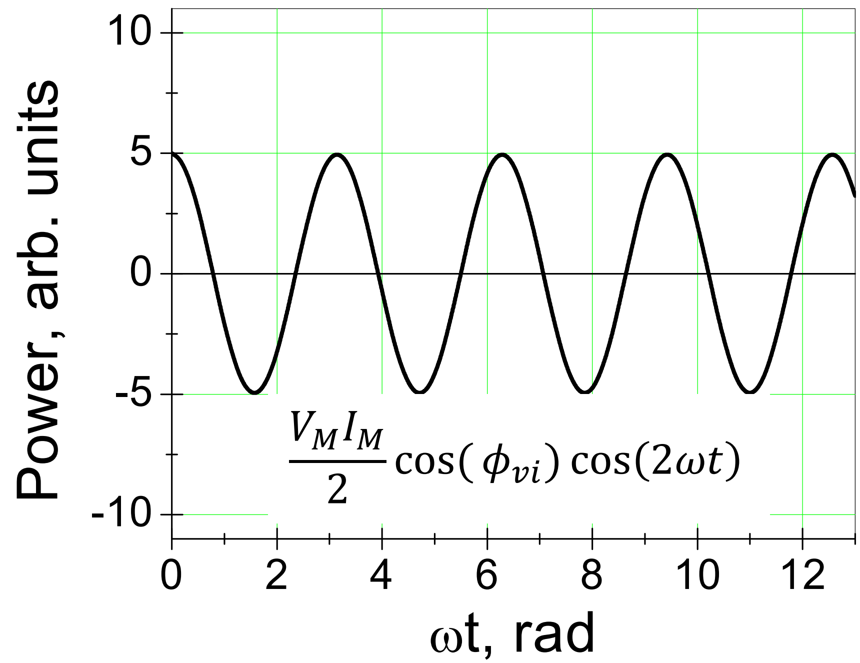
$$p(t) = \frac{V_M I_M}{2}\cos(\phi_{vi}) \cdot [1 + \cos(2\omega t)] - \frac{V_M I_M}{2}\sin(\phi_{vi})\sin(2\omega t)$$

where $\varphi_{vi} = \varphi_v - \varphi_i$ *is the phase shift between the voltage and the current*

Active (average) and reactive powers in R-L-C circuits

$$p(t) = \frac{V_M I_M}{2}\cos(\phi_{vi}) \cdot [1+\cos(2\omega t)] - \frac{V_M I_M}{2}\sin(\phi_{vi})\sin(2\omega t)$$





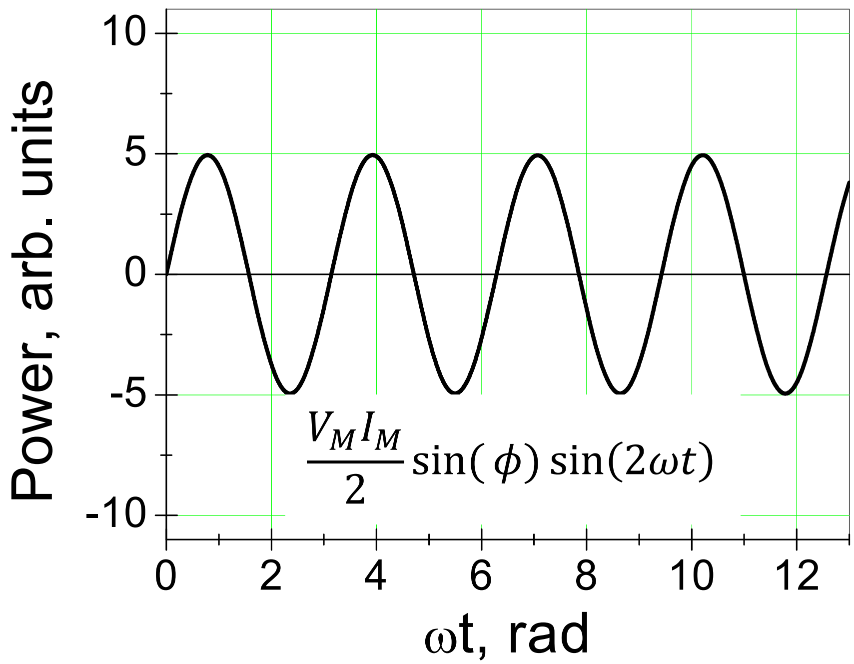
Active (average) power:

$$P = \frac{V_M I_M}{2}\cos(\varphi_{vi}) = V_{eff} I_{eff}\cos(\varphi_{v_i})$$

where $\varphi_{vi} = \varphi_v - \varphi_i$ is the phase shift between the voltage and the current

Active (average) and reactive powers in R-L-C circuits

$$p(t) = \frac{V_M I_M}{2}\cos(\phi_{vi}) \cdot [1 + \cos(2\omega t)] - \frac{V_M I_M}{2}\sin(\phi_{vi})\sin(2\omega t)$$



Reactive power:

$$Q = \frac{V_M I_M}{2}\sin(\varphi_{vi}) = V_{eff} I_{eff}\sin(\varphi_{vi})$$

where $\varphi_{vi} = \varphi_v - \varphi_i$

Active (average) and reactive powers in R-L-C circuits

$$p(t) = P \cdot [1 + \cos(2\omega t)] - Q \sin(\phi_{vi}) \sin(2\omega t)$$

$$P = \frac{V_M I_M}{2} \cos(\phi_{vi})$$

$$Q = \frac{V_M I_M}{2} \sin(\phi_{vi})$$

Examples:

1. Resistive circuit: $\varphi_{vi} = 0$;
$P = V_M I_M/2$; $Q = 0$;

2. Inductance: $\varphi_{vi} = -\pi/2$;
$P = 0$; $Q = -V_M I_M/2$;

2. Capacitance: $\varphi_{vi} = \pi/2$;
$P = 0$; $Q = +V_M I_M/2$;

Complex power, active (average) and reactive powers

Given the complex voltage and current amplitudes $\hat{V}_M = V_M e^{j\phi_v}$ $\hat{I}_M = I_M e^{j\phi_i}$

We define the Complex Power as $S = \frac{\hat{V}_M \cdot \hat{I}_M^*}{2} = \frac{V_M I_M}{2} e^{j(\varphi_v - \varphi_i)}$ " * " is "complex conjugate",

Rewriting S using Euler's formula:

$$S = \frac{V_M I_M}{2} e^{j(\varphi_v - \varphi_i)} = \frac{V_M I_M}{2} \cos(\varphi_{v_i}) + j \frac{V_M I_M}{2} \sin(\varphi_{vi})$$

Where we defined
$\varphi_{vi} = \varphi_v - \varphi_i$

Hence, real part of S = active power P
And imaginary part of S = reactive power Q

Complex power S using effective (rms) values

$$\hat{V}_{eff} = \hat{V}_M / \sqrt{2}; \hat{I}_{eff} = \hat{I}_M / \sqrt{2}$$

$$S = \hat{V}_{eff} \cdot \hat{I}_{eff}^*$$

Complex power, active (average) and reactive powers

In summary (formulae in red show MATLAB functions):

Complex power: $S = V_M * I^*_M/2 = V_M*conj(I_M)/2$

Active (Average) power: $P = Re(S) = real(S)$

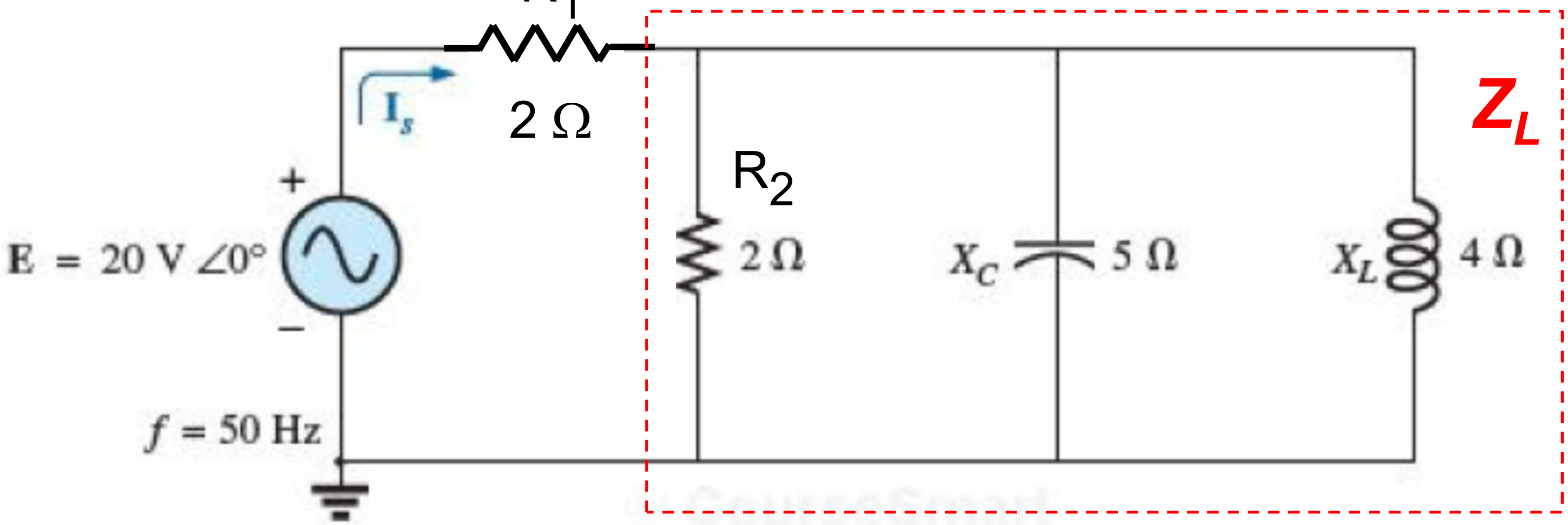
Reactive power, absolute value: $Q = |Im(S)| = abs(imag(S))$

Additional power characteristics (less common):

Apparent Power $|S| = abs(S) = V_M I_M/2$

Power factor $= P/|S| = \cos(\varphi_{vi})$

Example 1



(1) Find the average power P and reactive power Q delivered to elements R1, R2, C and L;

(2) Consider the R2-C-L combination as a load for the circuit and find the active and reactive power delivered to the load.

MATLAB code for the Example 1

```
clc
clear all
E=20;
R1=2; R2=2;
XL=4;XC=5;
ZL=j*XL;ZC=-j*XC;
%nodal analysis
Y=[1/R1+1/R2+1/ZL+1/ZC];
Is=[E/R1];
F=Y\Is;
%Voltage across R1
VR1=E-F;
%Current through R1
IR1=VR1/R1;
%Power in R1
SR1=VR1*conj(IR1)/2;
PR1=real(SR1)
QR1=imag(SR1)
```

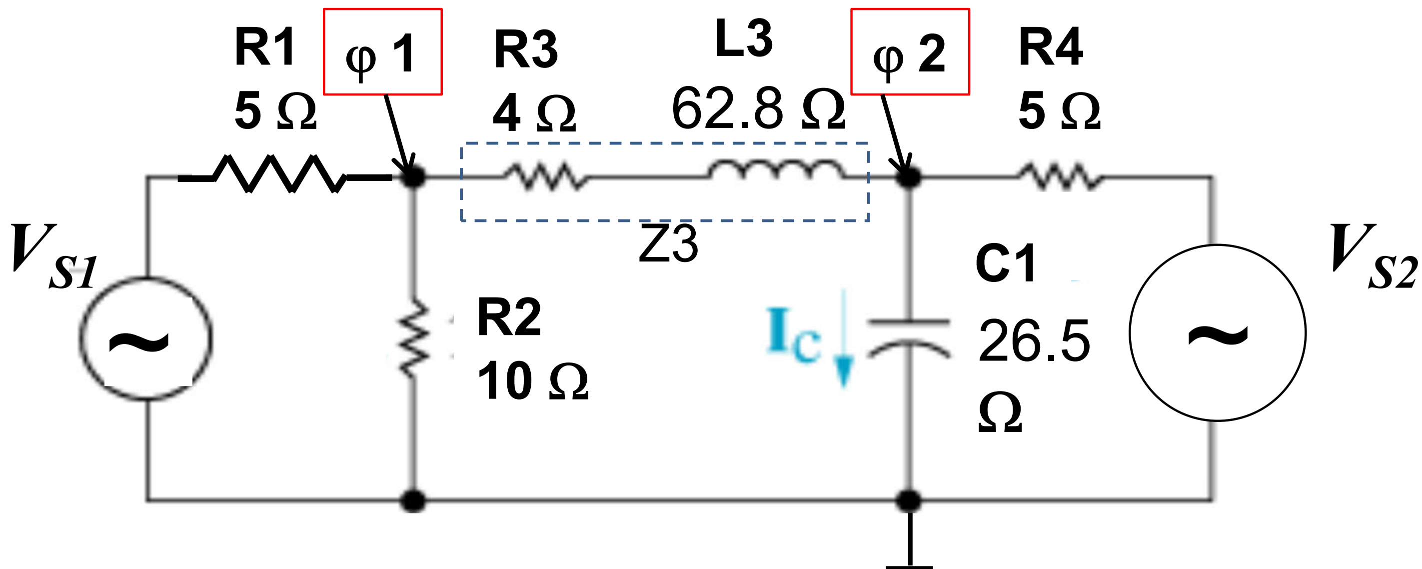
Continued in the right column

```
%Powers in R2,C and L
%Voltage across R2,C,L = F
IR2=F/R2;
IC=F/ZC;
IL=F/ZL;
SR2=F*conj(IR2)/2;
PR2=real(SR2)
QR2=imag(SR2)
SC=F*conj(IC)/2;
PC=real(SC)
QC=imag(SC)
SL=F*conj(IL)/2;
PL=real(SL)
QL=imag(SL)
%Power in the load ZL
%Voltage across ZL = F
ZL=1/(1/R2+1/ZC+1/ZL);
IZL=F/ZL;
SZL=F*conj(IZL)/2;
PZL=real(SZL)
QZL=imag(SZL)
```

Solution results:

PR1 = 25.1870
QR1 = 0
PR2 = 24.9377
QR2 = 0
PC = 0
QC = -9.9751
PL = 0
QL = 12.4688
PZL = 24.9377
QZL = 2.493

Example 2 (Lecture 11: AC nodal analysis)
Find the power dissipated in component Z3



$V_{S1}(t) = 7.5\cos(\omega t)$ V ⇨ $V_{S1} = 7.5$

$v_{S2}(t) = 20\cos(\omega t - 0.7\text{rad})$ V ⇨ $\mathbf{V_{S2}} = 20\, e^{j-0.7} = \mathbf{20\angle(-0.7)}$

ZL_3 = iXL_3;

Z3 = R3+ZL3;

ZC_1 = -iXC1;

$$Y = \begin{bmatrix} 1/R_1 + 1/R_2 + 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 + 1/Z\,C_1 + 1/R_4 \end{bmatrix}; \quad I_S = \begin{bmatrix} V_{S1}/R_1 \\ V_{S2}/R_4 \end{bmatrix};$$

[φ] = Y^{-1} × I_s =Y\\I_s ; **I_C = φ (2) /ZC_1**

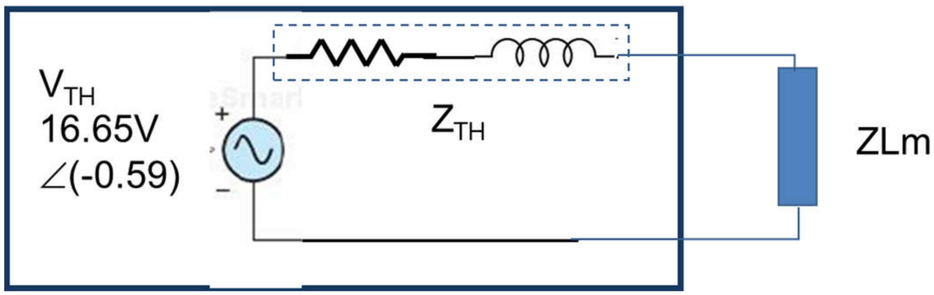
```
clear all
clc
% <<< AC Nodal analysis >>>
%Lect 17 Ex 1
%input data
Vs1m=7.5;
Vs2m=20; Vs2ph=-0.7;
R1=5;R2=10;R3=4;R4=5;
XL3=62.8; XC1=26.5;
%Complex amplutes and impedances
Vs1=Vs1m; %complex amplitude of Vs1(t)
Vs2=Vs2m*exp(j*Vs2ph);
ZL3=j*XL3;
ZC1=-j*XC1;
Z3=R3+ZL3;
%Y and Is matrices
Y=[1/R1+1/R2+1/Z3,-1/Z3;...
-1/Z3,1/Z3+1/ZC1+1/R4];
Is=[Vs1/R1;Vs2/R4];
Phi=Y\Is;
V3=Phi(1)-Phi(2);
I3=V3/Z3;
% <<< Power calculations >>>
S=V3*conj(I3)/2;
P=real(S) % active power in Z3
Q=abs(imag(S)) % abs. value of
                 % reactive power in Z3
```

Matlab output:

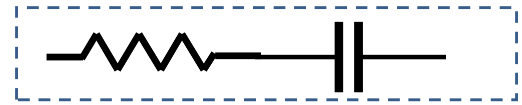
```
P =
   0.1443
Q =
   2.2654
```

Example 3
Find the active and reactive power dissipated in matching load

Zth = 1.38k +i5.08k



ZLm = 1.38k - i5.08k



```
clear all
clc
%AC Power in the matching load
%input data
Vthm=16.65; Vthph=-0.59;
Vth=Vthm*exp(i*Vthph);
Zth=1.38e3+5.08e3i;
ZLd=conj(Zth);
ILd=Vth/(Zth+ZLd);
VLd=ILd*ZLd;
S=VLd*conj(ILd) /2;
PLd=real(S);
QLd=abs(imag(S));
disp(['Active power in the load: PL= ',num2str(PLd), '  W'])
disp(['Reactive power in the load: QL= ',num2str(QLd), ' W'])
```

MATLAB output for this part

Active power in the load: PLd= 0.025111 W
Reactive power in the load: QLd= 0.092437 W

Example 3 (cont)

Find the physical equivalent circuit of the matching load; assume that f=2MHz

MATLAB code (continued from the previous slide):

```
%Equivalent circuit of the matching load at 2 MHz
f=2e6; om=2*pi*f;
%Resistance of matching load
RLd=real(ZLd);
disp(['Matching load contains resistance RLd= ',...
num2str(RLd),' Ohm in series with'])
% The sign of imag(ZLd)
ImZLd=imag(ZLd);
if ImZLd>0, %load contains inductor
XLLd=ImZLd; %equivalent inductor reactance XLd=om*LLd
LLd=XLLd/om;
disp(['inductor LLd= ', num2str(LLd),' H'])
else %load contains capacitor
XCLd=-ImZLd; %capacitor reactance XCLd=1/(om*CLd)
CLd=1/(om*XCLd);
disp(['capacitor CLd= ', num2str(CLd),' F'])
end
```

Output for this part

```
Matching load contains
resistance RLd= 1380 Ohm
in series with
capacitor CLd= 1.5665e-11 F
```